Course Name: Discrete Mathematics

CourseOutcomes(CO):

Analyze logical propositions via truth tables.

1. Prove mathematical theorems using mathematical induction.
2. Understand sets and perform operations and algebra on sets.
3. Determine properties of relations, identify equivalence and partial order relations, sketch relations.
4. Identify functions and determine their properties.
5. Define graphs, digraphs and trees, and identify their main properties.
6. Evaluate combinations and permutations on sets.

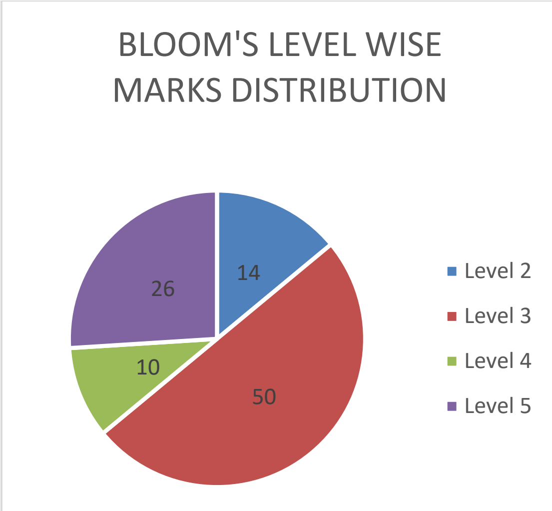
Question Paper
Total Duration (H:M): 3:00
Course: Discrete Mathematics
Maximum Marks: 100

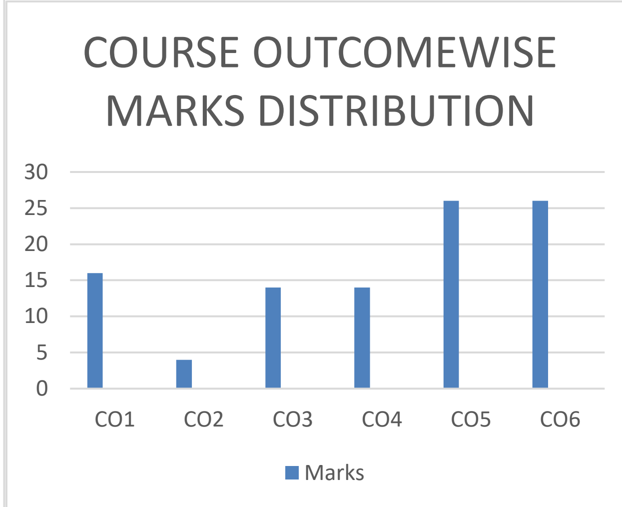
Note:*Attempt all questions.*

Q. No	Questions	Marks	CO	BL
1a)	Show that the relation "x R y x – y is divisibleby 3" where x, y Idefined in the set of integersI is an equivalence relation.	4	CO4	L3
1b)	(a)You have a fair die with 6 faces marked 1 to 6. You continue to roll the die repeatedly and only stop when either you roll a 1 or you voluntarily decide to stop at some point. When you stop you get a score that is equal to the value of the last roll. So, your last score is either 1 or the value of the last roll before you decided to stop. I. What stopping strategy will you choose to maximize your expected score.	6	CO6	L5
1c)	(a) Let A = {–2, –1, 0, 1, 2} and f:A Z (set ofintegers) be given by f(x) = x 2 – 2x – 3 find (a) the range of f, (b) pre-images of 6, –3, –5. (b) Let f: A B if function f is one-one onto, thenshow that f^-1 is also one-one onto.	10	CO4	L3
2a)	Prove the following identity in a Boolean algebra (B, +, . , ')	4	CO3	L2

	(a + b). (a' + c) = a.c + a'.ba.b, cB.			
2b)	Consider the statement "if the goods are unsatisfactory, then your money will be refunded". This was an advertising slogan of the T. Eaton Company. Is the given statement logically equivalent to "goods satisfactory or money refunded"? What about "if your money is not refunded, then the goods are satisfactory"? And what about "if the goods are satisfactory, then your money will not be refunded".	6	CO1	L2
2c)	(a) Draw the logic circuit for the following expression. f (a + b). (a' + b' + c'). (b'. c). (b) Draw a circuit for the following Boolean function and replace it by a simpler one: F (x, y, z) = x.z + [y. (y' + z). (x' + x.z')]	10	CO3	L3
3a)	A sign posted outside of Tokyo says "In order to attack the city, you must be green and related to Godzilla. If you are not green and not related to Godzilla, then you cannot attack the city". (a) Render the two statements on the sign in symbols. Start with: Let a be the assertion "you can attack the city", and carry on from there. (b) Argue that the two statements on the sign are not logically equivalent, contrary to what the author probably intended. Which is more restrictive on who can attack Tokyo? (c) Correct the second statement so that it is logically equivalent to the first one.	4	CO1	L5
3b)	Use known logical equivalences to do each of the following. (a) Show $p \rightarrow (q \lor r) \Leftrightarrow (p \land \neg q) \rightarrow r$. (b) Show $\neg(p \lor q) \lor (\neg p \land q) \lor \neg(\neg p \lor \neg q) \Leftrightarrow \neg(p \land \neg q)$. (c) Find an expression logically equivalent to $\neg(p \leftrightarrow q)$ that involves only $\neg$ and $\lor$.	6	CO5	L3
3c)	(a) Show that a complete graph with five vertices is not a planar graph. (b) Show that a simple graph with n vertices has N(N-1)/2 maximum number of edges. a.	10	CO6	L3
4a)	Assume that a and b are integers. Consider the statements: A: - If c is a prime number such c divides ab, then c divides a or c divides b. B: - If c is a prime number such c divides ab, and c does not divide b, then c divides a.	4	CO2	L4

	(a) Write the statements A and B in symbolic form and then show that they are logically equivalent. (b) Write the contrapositive of each statement in English.			
4b)	(a) A group of 8 scientists is composed of 5-psychologists and 3-sociologists, In how many ways can a committee of 5 be formed that has 3- psychologists and 2-sociologists. (b) How many ways can we distribute 14 indistinguishable balls in 4 numbered boxes so that each box is non empty	6	CO5	L5
4c)	(a) Prove that :(p q) (q r) (p r) is a tautology. (b) Show that: ~ (p q) p (~ q). (c) Explain the universal and existential quantifiers and also explain its negation.	10	CO5	L3
5a)	Show that the two statements $(p \land q) \rightarrow r$ and $(p \rightarrow r) \land (q \rightarrow r)$ are not logically equivalent.	4	CO5	L2
5b)	Find all combinations of truth values for p,q and r for which the statement $\neg p \leftrightarrow (q \land \neg(p \rightarrow r))$ is true.	6	CO1	L4
5c)	(a)LetG= (V, E) be a simple graph with n nodes. Let $u, v \in V$ be non-adjacent nodes such that $deg(u)+deg(v) \geq n$. Construct graph G= (V, E) by adding (u,v) to E- that is E=E+(u,v). Argue that if G has a Hamiltonian cycle, then G has a Hamiltonian cycle. (b)A bipartite graph G= ((X, Y), E) is degree constrained when $\exists d>0$ such that $deg(x) \geq d \geq deg(y)$ for all $x \in X$ And $Y \in Y$. i. Argue That: If G= ((X, Y), E) is a degree constrained bi-partite graph then it has a matching that saturates X. ii. A graph is regular when all nodes in the graph have the same degree. Supposing G is a regular, bi-partite graph then what can we say about a maximum matching of G? Justify your answer.	10	CO6	L5





BL–Bloom'sTaxonomyLevels(1-Remembering,2-Understanding,3 –Applying,4–Analyzing,5 –Evaluating,6-Creating)

CO–CourseOutcomes