

## Model Question Paper

COURSE: M.TECH.

BRANCH: POWER SYSTEMS

SEMESTER: 1

SUBJECT: ADVANCED MATHEMATICS

Duration: 3:00 hrs

Max marks: 100

**Note: Attempt all questions.**

**1. Attempt any four parts of the following:**

**5x4=20**

A. The eigen vectors of a  $3 \times 3$  real symmetric matrix  $A$  corresponding to the eigen values 2, 3, 6 are  $[1 \ 0 \ -1]^T$ ,  $[1 \ 1 \ 1]^T$  and  $[-1 \ 2 \ -1]^T$  respectively. Find the matrix  $A$ .

B. Solve the following equations by Gauss-Jordan method

$$10x + y + z = 12, \quad x + 10y + z = 12 \quad \text{and} \quad x + y + 10z = 12$$

C. Using Relaxation method solve the system of equations:  $9x - 2y + z = 50$ ,  $x + 5y - 3z = 18$  and  $-2x + 2y + 7z = 19$ .

D. Determine the largest eigen value and the corresponding eigen vector of the matrix  $A =$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

E. Using the iterative method, find the inverse of  $A = \begin{bmatrix} 1 & 10 & 1 \\ 2 & 0 & 1 \\ 3 & 3 & 2 \end{bmatrix}$  taking  $B = \begin{bmatrix} 0.4 & 2.4 & -1.4 \\ 0.14 & 0.14 & -0.14 \\ -0.85 & -3.8 & 2.8 \end{bmatrix}$

**2. Attempt any four parts of the following.**

**5x4=20**

A. Find a complete integral of  $pz = 1 + q^2$

B. Find a complete integral of  $z^2(p^2 + q^2) = x^2 + y^2$

C. Solve  $\frac{\partial^3 z}{\partial x^2 \partial y} - 2 \frac{\partial^3 z}{\partial x \partial y^2} + \frac{\partial^3 z}{\partial y^3} = \frac{1}{x^2}$

D. If a string of length  $l$  is initially at rest in equilibrium position and each of its point is given the velocity  $\left(\frac{\partial y}{\partial t}\right)_{t=0} = b \sin^3 \frac{\pi x}{l}$ , find the displacement  $y(x, t)$ .

E. A rectangular plate with insulated surface is  $10\text{cm}$  wide and so long compared to its width that it may be considered as infinite in length without introducing an appreciable error. If the temperature of the short edge  $y = 0$  is given by  $u = 20x$  for  $0 \leq x \leq 5$  and  $u = 20(10 - x)$  for  $5 \leq x \leq 10$  and the two long edges  $x = 0, x = 10$  as well as the other short edge are kept at  $0^\circ\text{C}$ , find the temperature  $u$  at any point  $(x, y)$ .

**3. Attempt any two parts of the following.**

**10x2=20**

- A. Given that  $I = Q = 0$  at  $t = 0$ , using Laplace transform, find  $I$  in the  $LR$  circuit for  $t > 0$  where  $E$  is the voltage (potential difference) and given by  $E = E_0 \sin \omega t$ .
- B. Using Fourier transformation solve  $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ ,  $0 < x < \pi$  subject to the conditions  $u(x, 0) = e^{-x}$ ,  $0 < x < \pi$ ,  $u(0, t) = 0$ ,  $u(\pi, t) = 0$ ,  $t \geq 0$ .
- C. Find the Fourier transform of the function  $f(x) = \begin{cases} 1 - x^2 & |x| < 1 \\ 0 & |x| > 1 \end{cases}$  hence evaluate

$$\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} \cos \frac{x}{2} dx$$

**4. Attempt any two parts of the following.**

**10x2=20**

- A. Find the Z- transform of  $c^k \sin ak$ ,  $k \geq 0$
- B. Solve  $y_{k+2} - \frac{5}{6}y_{k+1} + \frac{1}{6}y_k = 3^k$ ,  $y_0 = 0$ ,  $y_1 = 1$  using Z-transform method.
- C. Using convolution theorem find the inverse Z-transform of  $\frac{z.(z+1)}{(z-1)^3}$ .

**5. Attempt any two parts of the following.**

**10x2=20**

- A. Define negative binomial distribution and show that Poisson distribution is a limiting case of the negative binomial distribution.
- B. Find the moment generating function of the continuous normal distribution given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty \text{ and find its mean and variance.}$$

- C. Find the mean, variance and the coefficients  $\beta_1, \beta_2$  of the distribution

$$dF = kx^2 e^{-x} dx, \quad 0 < x < \infty$$